

Waves in the solar photosphere

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24 February 2008

ABSTRACT

The solar photosphere is a partially ionized medium with collisions between electrons, various metallic ions and neutral hydrogen playing an important role in the momentum and energy transport in the medium. Furthermore, the number of neutral hydrogen atom could be as large as 10^4 times the number of plasma particles in the lower photosphere. The non-ideal MHD effects, namely Ohm, Ambipolar and Hall diffusion can play an important role in the photosphere. We demonstrate that Hall is an important non-ideal MHD effect in the solar photosphere and show that Hall effect can significantly affect the excitation and propagation of the waves in the medium. We also demonstrate that the non-ideal Hall dominated inhomogeneous medium can become parametrically unstable, and it could have important ramification for the photosphere and chromosphere of the sun. The analysis hints at the possibility of solar photosphere becoming parametrically unstable against the linear fluctuations.

Key words: Sun: Photosphere, MHD, waves.

1 INTRODUCTION

The solar atmosphere is a partially ionized medium with fractional ionization ($X_e = n_e/n_n$) – a ratio of the electron (n_e) to the neutral (n_n) number densities, changing with the altitude. In fact, the lower solar atmosphere – the photosphere (≤ 500 km) is a weakly ionized medium where collisions between electrons, metallic ions and neutrals are responsible for the dynamical behaviour of the medium. With increasing altitude, the fractional ionization, which is 10^{-4} in the photosphere drops to 10^{-2} in the chromosphere at 1000 km. Even at an altitude $\sim (2000 - 2500)$ km, the neutral number density dominates the plasma density by an order of magnitude (Vernazza et al. 1981). Thus, the solar atmosphere is a partially ionised mixture of plasma and neutral gas with very weakly ionized gas in the photosphere and fully ionized plasma in the upper chromosphere and beyond.

Clearly, a partially ionized mixture of gas does not behave like fully ionized plasma and hence, ideal magnetohydrodynamics (MHD) description of the dynamics of solar photosphere at best is a very crude approximation. The investigation of Alfvén wave in the ideal MHD has been very popular topic in the solar physics owing to its appeal to the solar coronal heating (Ionson 1978; Priest 1987; Goedbloed & Poedts 2004). These waves can be easily excited

in the medium due to easy availability of vast reservoir of energy such as the convective gas motions. Only a tiny fraction of this energy, if carried by the wave to higher altitude, will suffice to heat the corona to high temperatures (Parhi et al. 1997). Implicit in this picture is the assumption that non-ideal MHD effects are unimportant in the photosphere. However, collision effects may severely affect the propagation of the wave in the solar atmosphere (De Pontieu & Haerendel 1998; Goodman 1998, 2004; Leake et al. 2005; Vranjes et al. 2007; Arber et al. 2007; Vranjes et al. 2008).

The importance of non-ideal MHD effect is related to the question of how well the magnetic field is coupled to the neutral matter. These effects can be quantified in terms of plasma–Hall parameter β_j

$$\beta_j = \frac{\omega_{cj}}{\nu_j}, \quad (1)$$

a ratio of the j^{th} particle cyclotron frequency $\omega_{cj} = eB/m_jc$ (where e, B, m_j, c denotes the electron charge, magnetic field, mass and speed of the light respectively) to the sum of the plasma-plasma, and plasma – neutral, ν_{jn} collision frequencies. For electrons $\nu_e = \nu_{en} + \nu_{ei} + \nu_{ee} \approx 2\nu_{en} + \nu_{ei}$ and for ions $\nu_i = \nu_{in} + \nu_{ii}$. It is clear that whenever $\beta_j \gg 1$, the j^{th} plasma particle will be strongly tied to the ambient magnetic field of the medium whereas in the opposite case, particles will not feel the Lorentz force and thus, can be treated as weakly magnetized or unmagnetized. For example, when ions and neutrals are strongly coupled, the relative drift of

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the “frozen-in” ions ($\beta_i \gg 1$) against the sea of neutrals may cause the diffusion of the magnetic flux – the so called ambipolar diffusion. When the ions and neutrals are moving together ($\beta_i \lesssim 1$), and there is relative drift between the electrons and ions (i.e. electrons are “frozen-in” the field but ions are not), the Hall diffusion¹ becomes important. When even electrons are not “frozen-in” the field, i.e. $\beta_e \lesssim 1$, the Ohmic dissipation becomes important. Therefore, the investigation of the non-ideal magnetohydrodynamic (MHD) effect in the solar atmosphere should be investigated in different parameter windows: (I) ambipolar – when the magnetic field can be regarded as frozen in the plasma and drifts with it through the neutrals (II) Ohmic – when neutrals stops the ionized particle from drifting with the field, and (III) Hall – when electrons are well coupled or partially coupled to the field and ions are partially or completely decoupled from the field. Since Hall parameter β_j will vary with the altitude, we shall anticipate that solar atmosphere will be in different non-ideal MHD regimes at different altitudes.

As noted above, non-ideal MHD effects are dependent upon the ambient physical parameters of the medium and their relative importance, and consequently, their role in the excitation and propagation of the waves can be gauged by the plasma-Hall parameter. In table 1, we give some relevant parameters for the solar photosphere (Vernazza et al. 1981) and corresponding plasma-Hall parameter. We have assumed that all ions are hydrogen ions although metallic ions generally dominate the cold photosphere. However, numbers for metal ions are speculative as the cross-section is not known. Assuming cross section $\sigma = 30 \times \sigma_{in}$ for the metallic ions ($m_i = 30 m_p$, here $m_p = 1.67 \times 10^{-24}$ g is proton mass) gives similar Hall beta parameters as for the ions (table 1). Therefore, we shall refrain from giving Hall parameter separately for the metallic ions. As is clear from the values of β_j at different altitude, the Ohmic dissipation will be dominant process at the surface of the photosphere. With the increasing altitude, Hall diffusion becomes the important mechanism for the flux distribution in the medium. Although numbers are not given for 250km, Hall starts to competes with the Ohmic diffusion at this altitude. For a 10^3 G field, Hall will dominate the Ohmic diffusion at the surface. The ambipolar diffusion is unimportant in the lower photosphere (≤ 1500 km) if $B = 10$ G, since $\beta_i \ll 1$. Only at higher altitude, typically between (1500 – 2500) km, ambipolar diffusion becomes important although Hall is only half as small as ambipolar in this region. Since neutral number density plummets rapidly beyond 2500 km, the role of ambipolar diffusion diminishes in the upper chromosphere. However, since cause of the Hall effect is due to the symmetry breaking between electrons and ions with respect to the magnetic field (Pandey & Wardle 2008), Hall will continue operating at this height as well albeit on a smaller scale. This conclusion appears counterintuitive since *Hall MHD in a fully ionized plasma* generally operates when the frequencies of interest is larger than the ion-cyclotron frequency over ion-inertial scale $\delta_i \equiv v_{Ai}/\omega_{ci}$, a ratio of the ion-

Alfvén speed ($v_{Ai} = B/\sqrt{4\pi\rho_i}$, $\rho_i = m_i n_i$ is the ion mass density) to the ion-cyclotron frequency. For a 10 G magnetic field, the cyclotron frequency is 10^5 , 10^6 and 10^7 s⁻¹ at $h = 0, 515$ and 1065 km respectively. Further the Hall scale, in a fully ionized plasma, $\delta_i \approx 2, 44$ and 71 cm, corresponding to $h = 0, 515$, and 1065 km respectively, is very small. Clearly, the *spatial and temporal scales of Hall MHD derived from a fully ionized, two component electron ion plasma suggest that the Hall effect is unimportant*. Therefore, fully ionized Hall MHD description of solar atmospheric plasma is irrelevant. Understandably, the role of Hall MHD of a fully ionized plasma in the solar atmosphere has been overlooked by the solar community.

The importance of Hall diffusion in angular momentum transport in protoplanetary discs (Wardle 1999; Balbus & Terquem 2001; Sano & Stone 2002a,b; Salmeron & Wardle 2003, 2005; Pandey & Wardle 2006) indicates that Hall operates on different spatial and temporal scales in a partially ionized plasmas. Recently, Pandey & Wardle (2008) developed a single fluid formulation for the partially ionized plasma and showed that *the spatio-temporal scale over which Hall operates in a partially ionized plasma is very different from a fully-ionized Hall-MHD*. Defining Hall frequency as (Pandey & Wardle 2008)

$$\omega_H = \frac{\rho_i}{\rho} \omega_{ci}, \quad (2)$$

where $\rho = \rho_n + \rho_i = m_n n_n + m_i n_i$ is the mass density of the bulk fluid with m_n, m_i as the mass and n_n, n_i as the number density of the neutral and ion fluids respectively, we can demand that if dynamical frequency of the system is larger than the Hall frequency, i.e. $\omega_H \lesssim \omega$, then Hall effect will be important. Furthermore, Hall in a partially ionized medium will not operate on the ion-inertial scale as in a fully ionized plasma, but over (Pandey & Wardle 2008)

$$L_H = \left(1 + \frac{\rho_n}{\rho_i}\right)^{(1/2)} \delta_i. \quad (3)$$

Assuming $m_i = m_n$, we see from Table 1 that Hall frequency varies between $10 - 10^5$ Hz and thus for fluctuations occurring at higher than ω_H , Hall will play an important role over $L_H \sim$ few hundred meters to few kms in the medium. We note that unlike high frequency Alfvén wave which damps in the solar medium (Leake et al. 2005), the excitation of low-frequency normal modes due to collisional coupling will propagate un-damped in the medium.

We investigate the propagation of waves in the solar photosphere in the Hall regime using recently developed general set of equations by Pandey & Wardle (2008) applicable to the partially ionized plasmas. The effect of dissipative diffusion on the wave properties has been investigated in the past (Tanenbaum & Mintzer 1962; Kulsrud & Pearce 1969). The wave dissipation in such a medium is dependent not only on the ion-neutral collision frequency but also on the fractional ionization of the medium (Kumar & Roberts 2003; Pandey & Wardle 2008). The wave damping have recently been studied in the context of spicules dynamics (Pontieu & Haerendel 1998). In the lower solar photosphere however, since Hall will dominate all other diffusive processes the resultant low frequency ion-cyclotron and collisional whistler wave will be the normal mode of the medium (Pandey & Wardle 2008).

¹ We note that the Hall drift could be a more appropriate terminology as diffusion is often associated with the dissipation which for Hall diffusion is zero. However, we shall use Hall diffusion to maintain the uniformity of the description.

Table 1. The neutral mass density $\rho = m_n n_n$, the ratio of ion to neutral mass density, the ion and electron collision frequencies and plasma Hall parameters at different heights of the solar atmosphere are given in the table. The ion mass $m_i = m_p$ and the magnetic field $B = 10\text{ G}$ has been assumed.

$h\text{ (km)}$	$n_n\text{ (cm}^{-3}\text{)}$	n_e/n_n	$\nu_{in}\text{ (Hz)}$	$\nu_{ii}\text{ (Hz)}$	$\nu_{en}\text{ (Hz)}$	$\nu_{ei}\text{ (Hz)}$	β_i	β_e
0	$1.2 \cdot 10^{17}$	$5.3 \cdot 10^{-4}$	$1.2 \cdot 10^9$	$2.2 \cdot 10^7$	$3.6 \cdot 10^{10}$	$1.3 \cdot 10^9$	10^{-5}	10^{-2}
515	$2.1 \cdot 10^{15}$	$1.2 \cdot 10^{-4}$	$1.8 \cdot 10^7$	$2.1 \cdot 10^5$	$5.2 \cdot 10^8$	$1.3 \cdot 10^7$	10^{-3}	10^0
1065	$1.7 \cdot 10^{13}$	10^{-2}	$1.7 \cdot 10^5$	$5.1 \cdot 10^4$	$5.1 \cdot 10^6$	$3.1 \cdot 10^6$	10^{-1}	10
1515	10^{12}	$6 \cdot 10^{-2}$	$1.1 \cdot 10^4$	$3.3 \cdot 10^4$	$3.2 \cdot 10^5$	$2.0 \cdot 10^6$	10^0	10^2
2050	$7.7 \cdot 10^{10}$	$5 \cdot 10^{-1}$	$8.8 \cdot 10^3$	$1.6 \cdot 10^4$	$2.6 \cdot 10^4$	$9.6 \cdot 10^5$	10^0	10^2
2543	10^9	$1.2 \cdot 10^0$	$0.9 \cdot 10^2$	$1.1 \cdot 10^0$	$2.6 \cdot 10^3$	$6.6 \cdot 10^1$	10	10^5

We investigate the linear and nonlinear wave properties of the medium in the present work. We show that since such a medium is inherently dispersive, the balance between the dispersion and nonlinearity leads to DNLS equation. The basic set of equations and the linearized dispersion relation is discussed in Sec. II. In Sec. III we first discuss the parametric instability and then describe the nonlinear equation. In Sec. IV discussion of the results and a brief summary is presented.

2 BASIC MODEL

The solar photosphere is a weakly ionized medium consisting of electrons, ions, neutrals. The single fluid description of such a plasma has been given in the past (Cowling 1957; Braginskii 1965). We shall use the single fluid description given by Pandey & Wardle (2008).

The continuity equation for the bulk fluid is given as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (4)$$

where $\rho = \rho_e + \rho_i + \rho_n \approx \rho_i + \rho_n$ is the bulk fluid density and \mathbf{v} is the bulk velocity, $\mathbf{v} = (\rho_i \mathbf{v}_i + \rho_n \mathbf{v}_n)/\rho$ with ρ_i , \mathbf{v}_i and ρ_n , \mathbf{v}_n as the mass density and bulk velocities of the ion and neutral fluids respectively. The momentum equation is given as

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla P + \frac{\mathbf{J} \times \mathbf{B}}{c}. \quad (5)$$

Here $\mathbf{J} = n_e (\mathbf{v}_i - \mathbf{v}_e)$ is the current density, \mathbf{B} is the magnetic field and $P = P_e + P_i + P_n$ is the total pressure. The induction equation is

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[(\mathbf{v} \times \mathbf{B}) - \frac{4\pi\eta}{c} \mathbf{J} - \frac{4\pi\eta_H}{c} \mathbf{J} \times \hat{\mathbf{B}} + \frac{4\pi\eta_A}{c} (\mathbf{J} \times \hat{\mathbf{B}}) \times \hat{\mathbf{B}} \right], \quad (6)$$

where $\hat{\mathbf{B}} = \mathbf{B}/B$, and the Ohmic (η), ambipolar (η_A) and Hall (η_H) diffusivity are

$$\eta = \frac{c^2}{4\pi\sigma}, \eta_A = \frac{D^2 B^2}{4\pi\rho_i \nu_{in}}, \eta_H = \frac{cB}{4\pi e n_e}. \quad (7)$$

Here $D = \rho_n/\rho$. We see from above Eq. (6) that the ratio of the Hall (H) and the Ohm (O) gives $H/O \sim \beta_e$ and the ratio between ambipolar (A) and Hall (H) gives $A/H \sim D^2 \beta_i$. In the photosphere, $D \rightarrow 1$, $\beta_i \ll 1$ and thus ambipolar diffusion can be neglected. In Fig.1, we show the ratio of A/H and H/O for the collision frequencies from table 1 for $B = 100\text{ G}$ field. The figure suggest

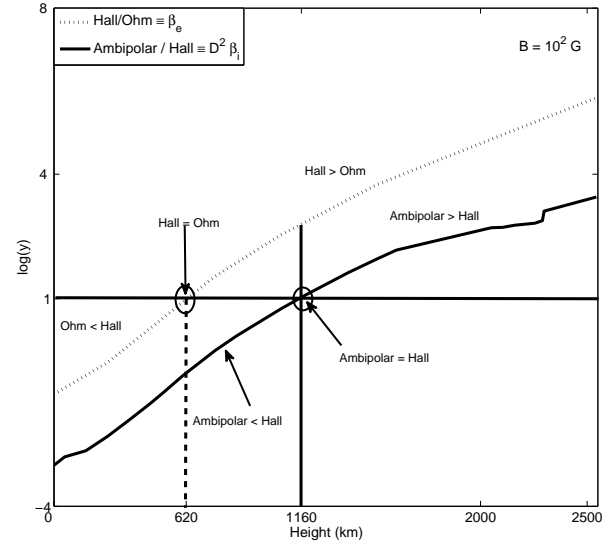


Figure 1. The ratio of Hall to Ohm (β_e), and ambipolar to Hall ($D^2 \beta_i$) is given for a $B = 10^2\text{ G}$ field.

that between $0 \leq h \leq 620\text{ km}$ in the solar atmosphere, Ohm will dominate both non dissipative Hall and dissipative ambipolar diffusion; between $620\text{ km} \leq h \leq 1160\text{ km}$ Hall dominates Ohm as well as ambipolar whereas between $1160\text{ km} \leq h \leq 2500\text{ km}$ ambipolar will dominate Hall. Clearly these estimates are based on the constant magnetic field strength. In Fig. 2, we show that ambipolar to Hall ratio changes with the changing field strength. For example, if $B = 10\text{ G}$, the Hall will dominate the ambipolar in $1000\text{ km} \leq h \leq 1600\text{ km}$ and only after 1600 km ambipolar becomes more important than Hall. Below 100 km Ohmic dissipation dominates all other form of diffusion. With increasing magnetic field strength, the domain of Ohmic dissipation shrinks and Hall starts operating at much closer to the surface. Therefore, we may say that for sufficiently strong field $B > 1\text{ kG}$, Hall diffusion will operate in the photosphere whereas ambipolar diffusion will operate in the chromosphere. Since Hall scale is dependent upon the fractional ionization of the medium (Pandey & Wardle 2008), with increasing altitude, the Hall scale $L_H = \eta_H/v_A$ will shrink and finally approach the ion inertial scale for a fully ionized plasma.

The role of collisional effects in the photosphere and chromosphere has been investigated in the past (Goodman

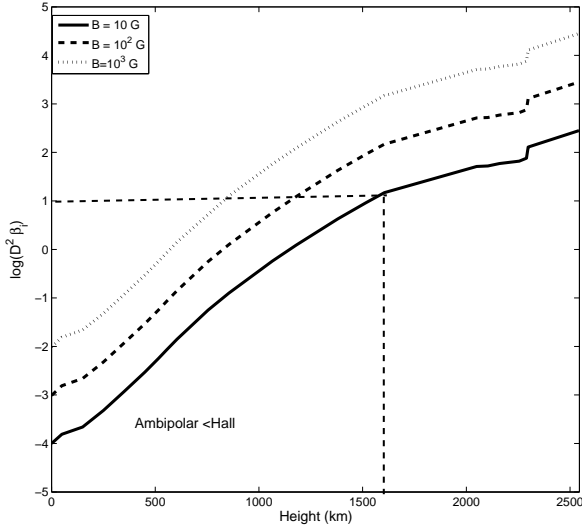


Figure 2. The ratio of ambipolar and Hall term is shown in this figure for varying magnetic field strength.

2004) using the conductivity tensor approach. We note that in the conductivity tensor approach plasma inertia is ignored while deriving generalized Ohm's law, and MHD waves (which require non-zero plasma inertia!) are studied using such a generalized Ohm's law. Clearly to overcome this logical inconsistency, a general set of equations Eqs. (4), (5), and (6) have been derived by Pandey & Wardle (2008). These set of equations can be closed by prescribing a thermodynamic relation between pressure and mass density. In order to compare the results of present formulation with the previous investigations e.g. (Goodman 2004), we note that the generalized Ohm's law $\mathbf{J} = \sigma_{\parallel} \mathbf{E}_{\parallel} + \sigma_{\perp} \mathbf{E}_{\perp} + \sigma_H \mathbf{E} \times \mathbf{B}/B$ (here $\mathbf{E} = \mathbf{E} + \mathbf{v} \times \mathbf{B}$ is the field in the bulk frame, $\mathbf{E}_{\parallel} = \mathbf{E} \cdot \hat{\mathbf{B}}$, $\mathbf{E}_{\perp} = \hat{\mathbf{B}} \times (\mathbf{E} \times \hat{\mathbf{B}})$ where $\hat{\mathbf{B}} = \mathbf{B}/B$) can be inverted to yield

$$\mathbf{E}' = \frac{\mathbf{J}}{\sigma_{\parallel}} - \left(\frac{\sigma_P}{\sigma_{\perp}^2} - \frac{1}{\sigma_{\parallel}} \right) \mathbf{J}_{\perp} - \frac{\sigma_H}{\sigma_{\perp}^2} (\mathbf{J} \times \hat{\mathbf{B}}). \quad (8)$$

Here $\sigma_{\perp} = \sqrt{\sigma_P^2 + \sigma_H^2}$ and $\mathbf{J}_{\perp} = \hat{\mathbf{B}} \times (\mathbf{J} \times \hat{\mathbf{B}})$. The parallel (σ_{\parallel}), Hall (σ_H) and Pedersen (σ_P) conductivities are given by Wardle & Ng (1999)

$$\sigma_{\parallel} = \frac{ce n_i}{B} (\beta_e + \beta_i), \quad (9)$$

$$\sigma_P = \frac{ce n_i}{B} \left(\frac{\beta_e}{1 + \beta_e^2} + \frac{\beta_i}{1 + \beta_i^2} \right), \quad (10)$$

$$\sigma_H = \frac{ce n_i}{B} \left(-\frac{\beta_e^2}{1 + \beta_e^2} + \frac{\beta_i^2}{1 + \beta_i^2} \right). \quad (11)$$

While writing above expressions for the conductivity tensors, we have assumed that plasma is quasineutral, i.e. $n_e \approx n_i$. Taking curl of Eq. (8) and using $c \nabla \times \mathbf{E} = -\partial_t \mathbf{B}$ we get the induction equation (6) except that η, η_H and η_A will be expressed as a combination of $\sigma_{\parallel}, \sigma_P$ and σ_H . Assuming $\beta_e \gg 1$ and $\beta_i \sim 1$, we note that

$$\hat{\sigma}_{\parallel} \approx \beta_e, \hat{\sigma}_P \approx \beta_i, \hat{\sigma}_H \approx 1, \quad (12)$$

where $\hat{\sigma} = B\sigma/(cen)$. In the $\beta_e \gg 1$ and $\beta_i \sim 1$ limit, the ratio of ambipolar and Hall terms from Eq. (8) can be written as

$$\frac{\left(\frac{\sigma_P}{\sigma_{\perp}^2} - \frac{1}{\sigma_{\parallel}} \right)}{\frac{\sigma_H}{\sigma_{\perp}^2}} \sim \beta_i, \quad (13)$$

which is same as derived from induction Eq. (6) except for the D^2 factor. This difference arises because in the conductivity tensor approach, ion inertia is set to zero and thus $D \equiv \rho_n/(\rho_i + \rho_n) = 1$ since $\rho_i = 0$. Clearly the induction equation of Pandey & Wardle (2008) is more general than the conductivity tensor approach which is valid only for weakly ionized plasma and is ill suited to describe the transition regions where plasma inertia is important. The ratio of Hall to Ohm from Eq. (8) becomes

$$\sigma_{\parallel} \frac{\sigma_H}{\sigma_{\perp}^2} \sim \beta_e, \quad (14)$$

which is the same as derived from eq. (6). Clearly in the weakly ionized limit Pandey & Wardle (2008) and conductivity approach gives identical results. It has been suggested in the past that Pedersen current dissipation could play an important role in the chromosphere (Goodman 2004). We see from Fig. 2 that ambipolar and Hall effect dominates the lower chromosphere (particularly for a strong magnetic field) and thus, the dissipative Pedersen current heating could be a plausible mechanism for coronal heating.

Eqs. (4), (5), and (6) together with an equation of state $P = c_s^2 \rho$ forms a closed set. We note that the collisional dissipation will cause the loss of energy in the lower solar atmosphere and therefore, generally a proper energy equation should be used for a more realistic modelling of the physical processes (Arber et al. 2007). However, in order to keep the description simple, we shall use $P = c_s^2 \rho$ to investigate the wave properties of the medium. In order to study waves in the photosphere, we shall assume a homogeneous background with no flow ($\mathbf{v}_0 = 0$). The equations (4), (5) and (6) after linearizing around $\mathbf{B}_0, P_0, \rho_0$ and Fourier analysing $\exp(i\omega t - i\mathbf{k} \cdot \mathbf{x})$ becomes

$$\omega \delta \rho - \rho \mathbf{k} \cdot \delta \mathbf{v} = 0, \quad (15)$$

$$\omega \delta \mathbf{v} = \frac{-1}{4\pi\rho} \left[(\mathbf{k} \cdot \mathbf{B}) \delta \mathbf{B} - \left(\frac{\omega^2}{\bar{\omega}^2} \right) (\delta \mathbf{B} \cdot \mathbf{B}) \mathbf{k} \right]. \quad (16)$$

Here $\bar{\omega}^2 = \omega^2 - k^2 c_s^2$. Defining $\hat{\mathbf{k}} \cdot \hat{\mathbf{B}} = \cos \theta$, along with Alfvén frequency $\omega_A = k v_A$, for $\delta \mathbf{B}$ we get

$$(\omega^2 - \omega_A^2 \cos^2 \theta) \delta \mathbf{B} = \frac{\omega^2}{\bar{\omega}^2} \omega_A^2 (\delta \mathbf{B} \cdot \hat{\mathbf{B}}) (\hat{\mathbf{B}} - \hat{\mathbf{k}} \cos \theta) - i k^2 \eta_H \omega \cos \theta (\hat{\mathbf{k}} \times \delta \mathbf{B}). \quad (17)$$

Only Hall and convective terms have been retained while linearizing the induction Eq. (6). After some straightforward algebra, following dispersion relation can be derived from equation (17)

$$\left(\omega^2 - \omega_A^2 \cos^2 \theta \right) \times \left(\omega^2 \left(1 - \frac{\omega_A^2}{\bar{\omega}^2} \sin^2 \theta \right) - \omega_A^2 \cos^2 \theta \right) = k^4 \eta_H^2 \omega^2 \cos^2 \theta. \quad (18)$$

Since compressional mode will not affect the transverse mode, we shall drop $k^2 c_s^2$ term in the above dispersion re-

lation. Then writing $k^2 \eta_H = \omega_A^2 / \omega_H$, we can recast above Eq. (18) as

$$\omega^2 = \omega_A^2 \cos^2 \theta \pm \left(\frac{\omega}{\omega_H} \right) \omega_A^2 \cos \theta. \quad (19)$$

This dispersion relation acquires a familiar form (cf. Wardle & Ng 1999, Eq. 25) when wave is propagating along the ambient magnetic field ($\theta = 0$)

In the $\omega_A \ll \omega_H$ limit, i.e. when the dressed ions (with effective mass $m^* = \rho/n_i$) gyrates faster than the time over which magnetic fluctuation propagates, the dispersion relation (18) gives the familiar Alfvén wave

$$\omega^2 \simeq \omega_A^2 \cos^2 \theta. \quad (20)$$

We note that these waves propagate in the neutral medium. The propagation of these low frequency Alfvén waves in a neutral medium occurs due to collisional coupling of the plasma particles with neutrals. The electrons, ions and neutrals are well coupled by the collisions and partake in the oscillations together.

The propagation of waves in a collision dominated medium appears counter intuitive since one would expect that collision will dissipate the energy and thus damping of the fluctuations will occur. This picture is valid only when wave frequencies are comparable or greater than the effective neutral-ion collision frequency, i.e. $k v_A \gtrsim \sqrt{\rho_n/\rho_i} (\beta_e/1 + D\beta_e) \nu_{ni}$ implying that the neutral is not hit often enough by the ions to participate in the oscillations (Pandey & Wardle 2008). Therefore, only fluctuations of certain wavelength will disappear due to collisional dissipation, and waves of wavelengths λ exceeding $\lambda_{\text{cutoff}} = \sqrt{2} \pi D v_A / \nu_{ni}$ can propagate in the medium. Since magnetic restoring force due to field deformation of wavelength λ acts only on the plasma particles, the neutrals at a distance λ apart can respond simultaneously to this restoring force only if the communication time between the neutrals ($\sim \lambda/v_A$) is smaller than the effective collision time $t_c \sim D \nu_{ni}^{-1}$. Only when t_c exceeds $\sim \lambda/v_A$, the wave in the medium will damp. Using table 1, we see that that since λ_{cutoff} is 0.01 cm at $h = 0$ and 7 m at $h = 1000$ km, the low frequency Alfvén wave of large wavelengths will propagate in the partially ionized solar photosphere without damping. Therefore we can say that the solar photosphere supports the excitation and propagation of the low frequency Alfvén waves in the predominantly *neutral* medium where the inertia of the fluid is carried by the neutral fluid and the magnetic deformation is felt by the plasma particles.

When $\omega_H \ll \omega_A$, i.e. when the dressed ion gyration period is slower than the propagation time of the magnetic fluctuations in the medium, the dispersion relation (18) can be analysed in low $\omega \ll \omega_A$ and high $\omega_A \ll \omega$ frequency limits. In the low frequency limit, we get

$$\omega^2 \simeq \omega_H^2 \cos^2 \theta. \quad (21)$$

In the high frequency $\omega_A \ll \omega$ limit,

$$\omega^2 \simeq \left(\frac{\omega_A^2}{\omega_H} \right)^2 \cos^2 \theta. \quad (22)$$

The mode described by Eq. (21) is the modified electrostatic ion cyclotron mode. Since $\omega \simeq \omega_H \sim 10 - 10^5$ Hz, solar photosphere can support ion cyclotron waves, except when the direction of wave propagation is almost transverse to

the ambient magnetic field. Equation (22) is the dispersion relation for the whistler mode. In the lower photosphere, the whistler is high frequency branch whereas ion-cyclotron frequency is the low frequency branch of the normal mode of the medium.

3 THE PARAMETRIC INSTABILITY AND THE SOLITONS

Assuming a uniform background field in the z-direction and the variation of the field along this direction only, above set of Eqs. (4), (5) and (6) can be written as

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial (\rho v_z)}{\partial z} &= 0, \\ \left(\frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z} \right) v_z &= -\frac{1}{\rho} \frac{\partial}{\partial z} \left(p + \frac{|B|^2}{8\pi} \right), \\ \left(\frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z} \right) v &= \frac{B_z}{4\pi\rho} \frac{\partial B}{\partial z}, \\ \frac{\partial B}{\partial t} &= -\frac{\partial}{\partial z} \left(v_z B - B_z v + \frac{i C_I \delta_H \rho_0}{\rho} \frac{\partial B}{\partial z} \right). \end{aligned} \quad (23)$$

Here $C_I = B_z / \sqrt{4\pi\rho_0}$, $\delta_H = v_A / \omega_H$ is the neutral skin depth, $C_I \delta_H = B_z \eta_H / B$, $B = B_x + i B_y$ and $v = v_x + i v_y$. A medium described by the above set of Eqs. (23) admits circularly polarized linear waves $(B, v) = (B_0, U_0) \exp i (\omega_0 t - k_0 z)$, as an exact solution. This can easily be seen by linearizing Eqs. (23) around a static equilibrium state. The linear dispersion relation provides a relation between the wave number k_0 and frequency ω_0 of the wave

$$\omega_0^2 = k_0^2 C_I^2 \left(1 \pm \frac{\omega_0}{\omega_H} \right). \quad (24)$$

The amplitudes B_0 and U_0 of the waves are related by

$$U_0 = - \left(\frac{B_0 \omega_0}{k_0 B_z} \right) \left(1 \pm \frac{\omega_0}{\omega_H} \right)^{-1}. \quad (25)$$

We note that the stability analysis of the steady-state background consisting of the unperturbed as well as the circularly polarized pump waves, i.e. $\mathbf{B} = (B_x(z), B_y(z), B_z)$ and $\mathbf{v} = (v_x(z), v_y(z), 0)$ with a constant density, indicates that the system is unstable, the so called modulational instability. The modulational instability has been extensively studied in the past (Hollweg 1974; Stenflo 1976; Derby 1978; Goldstein 1978; Brodin & Stenflo 1988; Pandey & Vladimirov 2007). Assuming such a steady-state background and linearizing Eqs. (23) one ends up with the following set of equations.

$$\begin{aligned} \frac{\partial \delta \rho}{\partial t} + \rho \frac{\partial \delta u_z}{\partial z} &= 0, \\ \frac{\partial \delta u_+}{\partial t} - \omega_0 \delta u_- &= \frac{B_z}{4\pi\rho} \left(\frac{\partial \delta B_+}{\partial z} + k_0 \delta B_- \right), \\ \frac{\partial \delta u_-}{\partial t} + \omega_0 \delta u_+ - k_0 U_0 \delta u_z &= \\ \frac{B_z}{4\pi\rho} \left(\frac{\partial \delta B_-}{\partial z} - k_0 \delta B_+ \right) + k_0 \frac{B_z B_0}{4\pi\rho^2} \delta \rho, \\ \frac{\partial \delta u_z}{\partial t} &= -\frac{C_s^2}{\rho} \frac{\partial \delta \rho}{\partial z} - \frac{B_0}{4\pi\rho} \frac{\partial \delta B_+}{\partial z}, \\ \frac{\partial \delta B_+}{\partial t} - \omega_0 \delta B_- &= B_z \left(\frac{\partial \delta u_+}{\partial z} + k_0 \delta u_- \right) - B_0 \frac{\partial \delta u_z}{\partial z} \end{aligned}$$

$$\begin{aligned}
& -\hat{\eta}_H \left(\frac{\partial^2 \delta B_-}{\partial z^2} - 2k_0 \frac{\partial \delta B_+}{\partial z} + k_0^2 \delta B_- \right) - \frac{\hat{\eta}_H k_0 B_0}{\rho} \frac{\partial \delta \rho}{\partial z}, \\
& \frac{\partial \delta B_-}{\partial t} + \omega_0 \delta B_+ = B_z \left(\frac{\partial \delta u_-}{\partial z} - k_0 \delta u_+ \right) + k_0 B_0 \delta u_z \\
& + \hat{\eta}_H \left(\frac{\partial^2 \delta B_+}{\partial z^2} + 2k_0 \frac{\partial \delta B_-}{\partial z} - k_0^2 \delta B_+ \right), \quad (26)
\end{aligned}$$

While writing the above set of equations, we have assumed that $\delta B_z = 0$. Here $\delta B_+ = \delta B_x \cos \phi + \delta B_y \sin \phi$, $\delta B_- = \delta B_x \sin \phi - \delta B_y \cos \phi$, $\delta u_+ = \delta u_x \cos \phi + \delta u_y \sin \phi$, $\delta u_- = \delta u_x \sin \phi - \delta u_y \cos \phi$, and $\hat{\eta}_H = B_z \eta_H / B$. Fourier analyzing the above set of equations (26), we will end up with following 8th order dispersion relation.

$$\omega^8 + a_7 \omega^7 + \dots + a_1 \omega + a_0 = 0. \quad (27)$$

The coefficients a_7, \dots, a_0 are given in the Appendix. Defining $\omega = \omega/\omega_0$, $x = k/k_0$, $b = \omega_0/\omega_H$, $F_{\pm} = 1/(1 \pm b)$, $\beta = C_s^2/C_I^2$, $k^2 C_s^2 = \omega_0^2 x^2 \beta F$, $a = B_0/B_z$ and noting that since the transition from stability to instability proceeds through $\omega = 0$, in the $\omega \rightarrow 0$ limit, we see that

$$\omega = -\frac{a_0}{a_1}. \quad (28)$$

In the long wavelength limit retaining only $\sim O(k)$ terms in the coefficients a_1 and a_0 one may write

$$\frac{a_1}{F_{\pm}^2} = a^2 [F_{\pm}^{-1} - (1+b)] - 2F_{\pm}^{-1} x, \quad (29)$$

$$\frac{a_0}{F_{\pm}^2} = - (F_{\pm}^{-2} + a^2) [1 - F_{\pm} (b+1)]. \quad (30)$$

Since for the left-circularly polarized pump waves, $F_+ = 1/(1+b)$, from Eq. (30), $a_0 = 0$. Thus in view of Eq. (28), we shall anticipate that the instability should decrease in the vicinity of $k = 0$. The numerical solution of the dispersion relation (27) for the left circularly polarized (Fig. 3(a)) indeed displays this feature. The normalized growth rate is plotted against the normalized wave number for $\omega_0/\omega_H = 0.01, 0.1$ and 0.15 . We see that the growth rate decreases in the vicinity of $k \rightarrow 0$. With the increase in ω_0/ω_H , the growth rate increases. As is clear from the wave dispersion relation (24), the increase in ω_0/ω_H implies the increasing importance of the Hall term ($\mathbf{J} \times \mathbf{B}$), which appears due to the relative drift between the plasma and the neutral (i.e. ion Hall- $\beta_i \leq 1$). This drift is caused entirely by the collisional coupling of the ions with the neutrals. Therefore, the increase in ω_0/ω_H implies the enhanced availability of the free energy reservoir. Hence with the increasing ω_0/ω_H , growth rate increases.

In Fig. 3(b) we plot the real (dotted line) and imaginary (solid line) part of the frequencies for $\omega_0/\omega_H = 0.01$. We see from Fig. 3(b) that the growth rate of instability causes the sharp decrease of the real frequency. It suggests that most of the available free energy of the pump is invested in the growth of the fluctuations resulting in the decay of the real part of the frequency. However, with the decrease in the growth rate, most of the pump energy goes to the real part of frequency, as seen from the figure. Further, we note that before the real part of frequency shows signs of recovery, the imaginary part start to fluctuate and finally drops to zero.

In Fig. 4 the growth rate for the left circularly polarized wave is plotted against the wave number, for different values

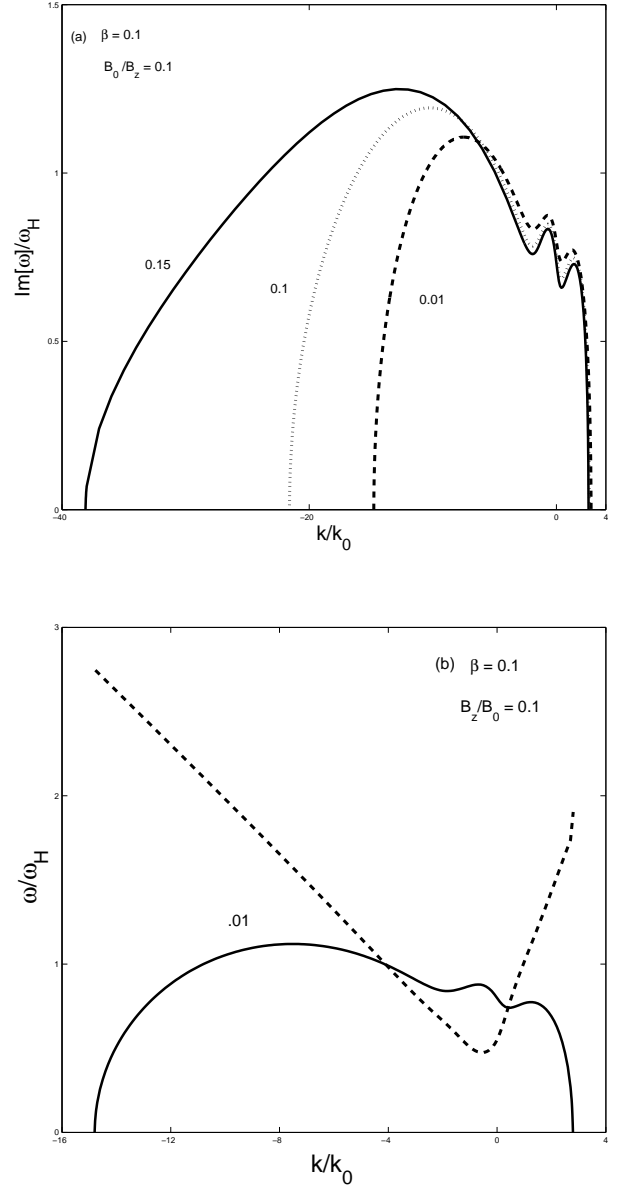


Figure 3. The growth rate of the left circularly polarized wave $Im[\omega]/\omega_H$ against k/k_0 is shown in Fig. 3(a) for $\omega_0/\omega_H = 0.01, 0.1$ and 0.15 with plasma $\beta = 0.1$ and $B_0/B_z = 0.1$. In Fig. 3(b) both real (dotted line) and imaginary (solid line) part of the frequency is given for $\omega_0/\omega_H = 0.01$. All other parameters are same as in Fig. 3(a).

of plasma beta, $\beta = 0.01, 1$, and 5 . Although the growth rate is dependent upon the plasma compressibility, the dependence is not linear. For example, when plasma β increases from 0.01 to 5 , the growth rate almost doubles. However, when both acoustic and Alfvén speed equals, i.e. $\beta = 1$, the growth rate is highest. For $\beta = 10$, the growth rate is similar to the case when $\beta = 5$. Therefore, the highest growth rate is when $v_A \approx c_s$. The growth rate is not very sensitive to the variation of the pump amplitude, B_0/B_z and hence is not given here.

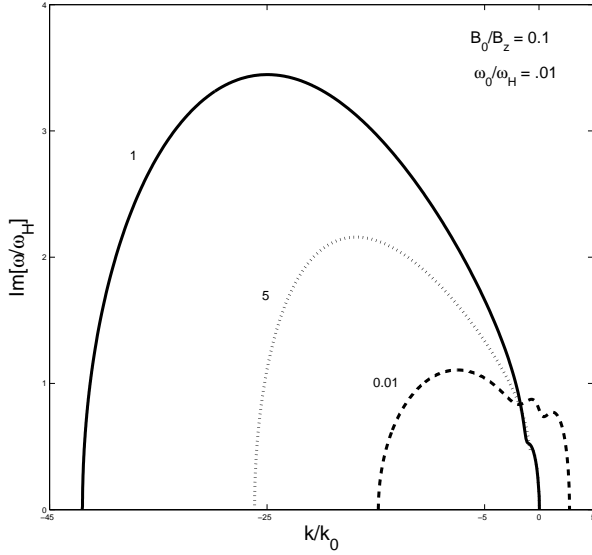


Figure 4. Same as in Fig. 1, with varying plasma β .

For the right-circularly polarized waves, when $F_- = 1/(1 - \omega_0/\omega_H)$, the waves can grow at a rate

$$Im[\omega] \approx F_-, \quad (31)$$

in the vicinity of $x = 0$. Since the growth rate (Eq. 31) is inversely proportional to $(1 - \omega_0/\omega_H)$, near $\omega_0 \simeq \omega_H$, when wave frequency matches the Hall frequency, the instability can grow resonantly. Therefore, the growth of the instability is quite different for the left and right-circularly polarized waves. Whereas for the left-circularly polarized waves, the instability decreases in the neighbourhood of $k = 0$, for the right-circularly polarized pump, the instability may become large near $k = 0$. However, for the right circularly polarized waves, unbounded growth of the instability is not possible since linear approximation will break down once fluctuation becomes comparable to the background equilibrium quantities.

In Fig. 5(a) the growth rate and in Fig. 5(b) corresponding real part of the frequency is shown for the right-circularly polarized waves. When $\omega_0/\omega_H = 0.9$, the growth rate becomes very large. The free energy is resonantly pumped into the fluctuations with increasing ω_0/ω_H . The physical system behaves more like a driven oscillator. The resonant driving is indirectly related to the neutral-plasma collisions. The relative drift between the plasma and the neutral causes a Hall field over the Hall time t_H ($t_H \equiv \omega_H^{-1}$). If the Alfvén wave propagation time ω_0^{-1} becomes comparable to the Hall time t_H , the energy is freely fed to the fluctuation by the pump to the plasma particles. Resulting free energy causes the large growth rate. This behaviour can be seen from the analytical expression Eq. (31). It should be noted from the corresponding curve in Fig. 5(b) that the real frequency shows a sharp decline for $\omega_0/\omega_H = 0.9$. This suggests that almost all the free pump energy is resonantly used in the fluctuation growth. Similar behaviour is also noted for $\omega_0/\omega_H = 0.95$. Since the instability growth rate in this case is larger than when $\omega_0/\omega_H = 0.9$, the part of the available free energy

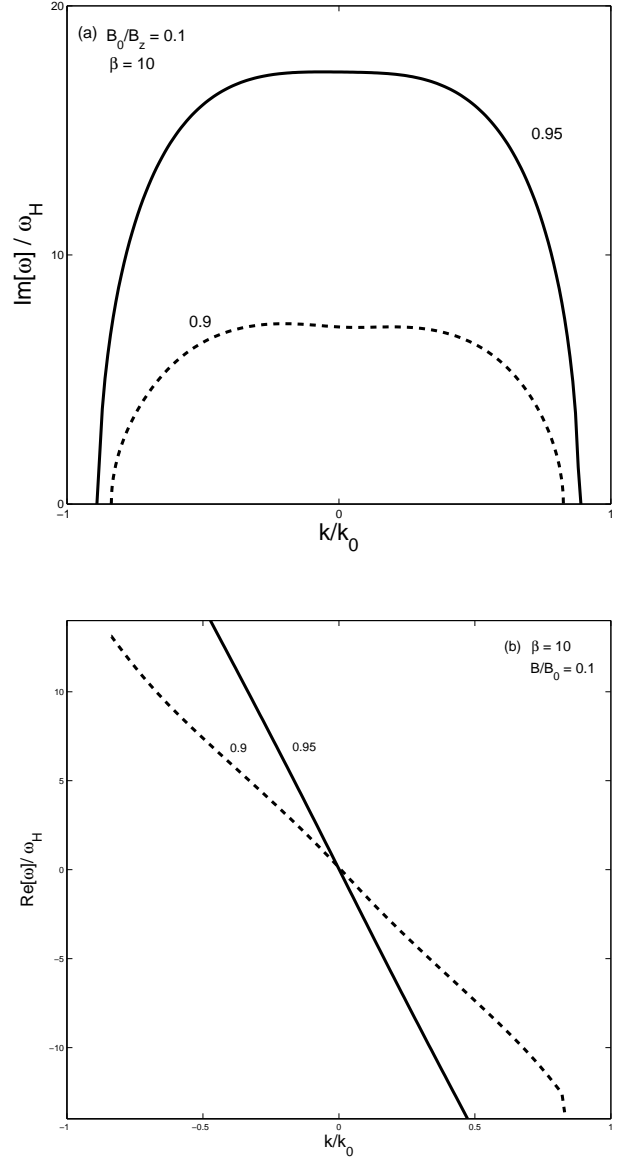


Figure 5. The dependence of the growth rate on the amplitude of the right handed circularly polarized pump wave is shown in Fig. 5(a) and corresponding real part of the frequency is shown in Fig. 5(b) for $\omega/\omega_H = 0.9$, and 0.95 . The plasma $\beta = 10$ and $B_0/B_z = 0.1$.

drains faster from the real part of the frequency. This is in agreement with the well known behaviour of the oscillators near resonance although present system is more complex.

It is believed that the granulations or convective motions are responsible for the Alfvén wave generation in the photosphere. However the medium is capable of exciting very low frequency fluctuations only, i.e. when the wave frequency is much smaller than the collision frequency. Then the neutrals are dragged along by collisions and as a result, they participate in the collision (Tanenbaum & Mintzer 1962). However, excitation of the high-frequency ideal MHD Alfvén mode is very unlikely. The high-frequency ideal MHD mode will be damped by collisions of plasma particles with

the neutrals as well as collisions of plasma particles with each other (Tanenbaum & Mintzer 1962; Petrovic et al.; Vranjes et al. 2007, 2008).

The excitation of very low frequency Alfvén wave is a promising candidate for heating and acceleration of the solar plasma from coronal holes. Taking $\omega_H = 1$ Hz for $\omega_{ci} \sim 10^4$ Hz we see that the growth rate of the left-circularly polarized wave $0.3\omega_0$ suggests that the instability of the Alfvén wave could be relevant to the turbulent heating. Since $\omega_H = 10 - 10^5$ Hz, the resonance condition for the right circularly polarized wave implies that ω_0 must be in the same range. Therefore, it is quite likely that the low frequency right circularly polarized waves may resonantly extract energy from the ambient surrounding.

4 DISCUSSION AND SUMMARY

The solar photosphere is a weakly ionized gas with neutral hydrogen carrying the inertia of the fluid. We show that the non-ideal effects are of paramount importance for the excitation and propagation of the waves. The hydromagnetic waves in such a medium are an outcome of the balance between the inertia of the neutral component with the deformation of the magnetic field, to which neutrals are tied only by the ion-neutral collisions. Therefore, very low frequency Alfvén wave can be excited in such a medium. The high frequency Alfvén mode (in the ion fluid) will be damped in such a medium (Vranjes et al. 2007, 2008).

The Hall diffusion dominates the ambipolar and Ohmic diffusion in photosphere. The Hall scale $L_H = (\rho/\rho_i)^{1/2} \delta_i$, is typically two order of magnitude larger than the usual ion-inertial length $\delta_i = v_{Ai}/\omega_{ci}$ (Pandey & Wardle 2008). The presence of the Hall effect will cause the excitation of the ion-cyclotron and whistler waves in the medium.

We show that in an inhomogeneous photosphere, circularly polarized whistlers, are an exact solution of the ensuing equations. These waves can act as a pump wave. The resulting parametric instability can make the medium turbulent.

We note that derivative nonlinear Schrödinger equation (DNLS) equations can be derived from Eqs. (23) using reductive perturbation technique (Kennel et al. 1988)

$$\frac{\partial b_\perp}{\partial \tau} + \alpha \frac{\partial}{\partial \xi} [(|b_\perp|^2 - |b_{\perp 0}|^2) b_\perp] + \frac{i \hat{\eta}_H}{2} \frac{\partial^2 b_\perp}{\partial \xi^2} = 0, \quad (32)$$

where $b_\perp = B/B_z$ and $\alpha = v_A C_I/4 (C_I^2 C_s^2)$. One can show that Eq. (32) admits magnetically compressive (bright) and magnetically depressive (dark) solitons (Hada et al. 1998). Defining $S_{1,2} = (b_0 \mp \sqrt{2\epsilon/\alpha})^2$ with $\epsilon = (V/v_A - C_I)/C_I$, describing the shift of the travelling wave speed V from the intermediate speed and $\eta = \xi - \epsilon\tau$ where $\xi = x - C_I t$ and $\tau = C_I t$, the solutions can be written as

$$\begin{aligned} S &= S_0 + \frac{S_2 - S_0}{1 + T_B \sinh^2(\alpha K \eta)}, \\ \Phi &= 3 \arctan(C_B \tanh(\alpha K \eta)) + \\ &\quad \text{sgn}\left(\frac{S_0}{2} - \frac{\epsilon}{\alpha}\right) \arctan(C_B^* \tanh(\alpha K \eta)), \\ T_B &= \frac{S_2 - S_1}{S_0 - S_1}, C_B = \sqrt{\frac{S_2 - S_0}{S_0 - S_1}}, \end{aligned} \quad (33)$$

and $C_B^* = \sqrt{S_1/S_2} C_B$. Here $0 < S_1 < S_0 < S_2$.

$$\begin{aligned} S &= S_0 - \frac{S_0 - S_1}{1 + T_D \sinh^2(\alpha K \eta)}, \\ \Phi &= -3 \arctan(C_D \tanh(\alpha K \eta)) \\ &\quad - \text{sgn}\left(\frac{S_0}{2} - \frac{\epsilon}{\alpha}\right) \arctan(C_D^* \tanh(\alpha K \eta)), \\ T_D &= \frac{S_2 - S_1}{S_2 - S_0}, C_D = \sqrt{\frac{S_0 - S_1}{S_2 - S_0}}, \end{aligned} \quad (34)$$

and $C_D^* = \sqrt{S_2/S_1} C_D$ with $K = 0.5 \sqrt{(S_2 - S_0)(S_0 - S_1)}$. The value $b_1 = \sqrt{S_1}$ represents the minimum of the transverse field magnitude in a refractive, dark soliton and $b_2 = \sqrt{S_2}$ is the peak value of b_\perp in a compressive, bright soliton. The magnetic field depression in dark maximizes when $b_1 = 0$, i.e. when transverse components b_y and b_z vanish. The magnitude of the field inside the dark soliton is determined by $B_z = B_0 \cos \theta$, implying that a large propagation angle will cause large magnetic cavities. The electric field for the magnetic cavities can be estimated as

$$E \sim B_0 v_A \left(b_0 \mp \sqrt{\frac{2\epsilon}{\alpha}} \right) / c \sin \theta. \quad (35)$$

Here c is speed of light. Such a field is available for the charged particle acceleration. We speculate that if such a soliton induced field indeed exists in the solar photosphere, then with increasing height, since Alfvén speed will increase from $5.4 \times 10^3 \text{ cms}^{-1}$ at $h = 0$ in the photosphere to $4 \times 10^5 \text{ kms}^{-1}$ at $h = 10^3 \text{ km}$ for a 10 G magnetic field, the electric field E will vary typically between 0.15 V/m to 1.2 V/m. Such a field may not be sufficient for accelerating the particles. However, if the field strength is increased by two to three orders of magnitude (as could be the case in the sunspots and pores) a million Volt field can easily exist that will be available for the acceleration and subsequent heating. We do not know of any observational support for the existence of soliton in the photosphere at this stage and thus the hypothesis of soliton generated field and ensuing charged particle acceleration should be treated with caution.

The following are the itemized summary of the present work.

- (1) Non-ideal MHD effect, namely Hall diffusion is important in the solar atmosphere.
- (2) The circularly polarized whistlers can be easily excited in the medium.
- (3) The photosphere can become parametrically unstable.
- (4) The nonlinear derivative Schrödinger equation describes the finite amplitude fluctuations. We speculate that the dark and bright soliton solutions may cause the particle acceleration in the medium.

ACKNOWLEDGMENTS

BP wishes to thank Mark Wardle for his constant encouragement and support. The financial support of Australian Research Council and Macquarie University grant is acknowledged. It is our pleasure to thank the organisers of Kodai-Trieste Plasma Astrophysics workshop (August, 27 September, 07), Kodaikanal, India which provided the stimulating atmosphere for developing part of this work.

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APPENDIX A: THE COEFFICIENTS OF THE DISPERSION RELATION EQ. (18)

Defining $b = \omega_0/\omega_H$, $a = B/B_z$ and $x = k/k_0$, the coefficients of dispersion relation are given as

$$\begin{aligned} a_8 &= 1, \quad a_7 = 3b F_{\pm} x, \\ \frac{a_6}{F_{\pm}} &= b(1+x^2) [-1 + b F_{\pm} (1-x^2)] - 2(1+x^2) - \frac{2}{F_{\pm}} \\ &\quad - x^2 (\beta + a^2 - 2b^2 F_{\pm}), \\ \frac{a_5}{x F_{\pm}} &= 2b F_{\pm} (1+x^2) + (a^2 + 2) [1 - b F_{\pm} (1-x^2)] \\ &\quad + 1 - a^2 - 3b - b F_{\pm} x^2 (\beta + a^2) - b [1 + F_{\pm} (1+x^2)] \\ &\quad + \frac{(1+x)}{x} - 2b [F_{\pm} (1+x^2) + 1 + F_{\pm} x^2 (\beta + a^2)] . \quad (A1) \end{aligned}$$

$$\begin{aligned} \frac{a_4}{F_{\pm}} &= (\beta + a^2) x^2 + F_{\pm} x^2 [\beta (1+x^2) + a^2] + a^2 x^2 \\ &\quad + 2bx [F_{\pm} x (1-a^2) - bx F_{\pm} - bx^3 F_{\pm}^2 (\beta + a^2)] \\ &\quad + bx F_{\pm} (1+x-2bx) - 2x^2 F_{\pm} (a^2 + 2) \\ &\quad + b(1+x^2) [1 - b F_{\pm} (1-x^2) - F_{\pm} (1+x^2)] \\ &\quad + [1+x^2 + F_{\pm}^{-1} + x^2 (\beta + a^2)] [1 + F_{\pm} (1+x^2)] \\ &\quad - [1+x^2 (1-a^2) - b(1+x^2) + F_{\pm}^{-1} - b\beta F_{\pm} x^2] \\ &\quad \times (1+x^2) [1 - b F_{\pm} (1-x^2)] . \quad (A2) \end{aligned}$$

$$\begin{aligned} \frac{a_3}{F_{\pm}^2} &= x (a^2 + 2) [-F_{\pm}^{-1} + b(1-x^2) + (1+x^2)] + 2x \\ &\quad [1+x^2 (1-a^2) - b(1+x^2) (1+\beta F_{\pm} x^2) + F_{\pm}^{-1}] \\ &\quad - [F_{\pm}^{-1} - b(1-x^2)] [F_{\pm} x^3 (2\beta + a^2) + 2x a^2] \\ &\quad + x^3 (\beta + a^2) (b-1) + 2bx [x^2 (\beta + a^2) + \beta F_{\pm} x^2 \\ &\quad \times (1+x^2) + a^2 x^2 (F_{\pm} + 1)] - x [F_{\pm}^{-1} + 1 + x^2] \\ &\quad \times [1 - b - a^2 - b F_{\pm} x^2 (\beta + a^2)] + [2bx - 1 - x] \\ &\quad \times [1+x^2 + F_{\pm}^{-1} + x^2 (\beta + a^2)] . \quad (A3) \end{aligned}$$

$$\begin{aligned} \frac{a_2}{F_{\pm}^2} &= [F_{\pm}^{-1} - b(1-x^2) - 1 - x^2] \\ &\quad [1+x^2 (1-a^2) - b(1+x^2) (1+\beta F_{\pm} x^2) + F_{\pm}^{-1}] \\ &\quad + 2x^2 [F_{\pm} x^2 (2\beta + a^2) + 2a^2] + [1 - b F_{\pm} (1-x^2)] \\ &\quad \times [F_{\pm}^{-2} - b\beta x^2 (1+x^2) + x^2 (\beta (1+x^2) - a^2)] \\ &\quad + 2b F_{\pm} x^4 (b-1) (\beta + a^2) - x^2 [F_{\pm}^{-1} + 1 + x^2] \\ &\quad \times [\beta + a^2 + F_{\pm} (\beta (1+x^2) + a^2) + a^2] \\ &\quad - x [2bx - 1 - x] [1 - a^2 - b - b F_{\pm} x^2 (\beta + a^2)] . \quad (A4) \end{aligned}$$

$$\begin{aligned} \frac{a_1}{F_{\pm}^2} &= [F_{\pm}^{-1} - b(1-x^2) - 1 - x^2] \times \\ &\quad [x^2 (2\beta + a^2) + 2a^2] - 2F_{\pm} x [F_{\pm}^{-2} - b\beta x^2 (1+x^2) \\ &\quad + x^2 (\beta (1+x^2) + a^2)] - x^3 (b - F_{\pm} x) (\beta + a^2) \\ &\quad \times [1 + F_{\pm} (1+x^2)] + x^2 (1+x-2bx) \times \\ &\quad [\beta + a^2 + F_{\pm} (\beta (1+x^2) + a^2) + a^2] . \quad (A5) \end{aligned}$$

$$\begin{aligned} \frac{a_0}{F_{\pm}^2} &= F_{\pm} x^3 (b-1) (\beta + a^2) (1+x-2bx) \\ &\quad - [F_{\pm}^{-2} - b\beta x^2 (1+x^2) + x^2 (\beta (1+x^2) + a^2)] \\ &\quad \times [1 - b F_{\pm} (1-x^2) - F_{\pm} (1+x^2)] . \quad (A6) \end{aligned}$$